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FATIGUE CHARACTERIZATION OF COMPOSITE MATERIALS

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Mechanics and Surface Interactions Branch
Nonmetallic Materials Division

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A procedure is outlined which allows the generation of an S-N curve with some statistical value without resorting to an extremely large data base. The procedure is based on a power law representation of the S-N curve and a two-parameter Weibull Distribution for time-to-failure at a specific stress level. A data pooling scheme is also discussed which allows the determination of a fatigue shape parameter which is independent of stress level.

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FOREWORD

In this report, a procedure is outlined which allows the generation of an S-N curve with some statistical value without resorting to an extremely large data base.

This report was prepared in the Mechanics and Surface Interactions Branch (AFML/MBM), Nonmetallic Division, Air Force Materials Laboratory, Wright-Patterson AFB, Ohio. The work was performed under Project 2419, "Nonmetallic Structural Materials," Task No. 241903, "Composite Materials and Mechanics Technology." The time period covered by this effort was from 1 October 1978 to 15 April 1979. James M. Whitney (AFML/MBM) was the laboratory project engineer.

The author wishes to acknowledge Professor W. Park of Wright State University for helpful suggestions concerning statistical analysis and G. Sendeckyj of the Air Force Flight Dynamics Laboratory for useful discussions concerning "strength degradation" and "wearout" models.

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SECTION I

INTRODUCTION

The classical S-N curve has been the primary method of characterizing the fatigue behavior of fiber reinforced composites. This method usually consists of determining the number of cycles to failure for a number of maximum stress ranges associated with a particular load history (often constant amplitude tension-tension loading). The resulting S-N curve yields an estimate of the mean time-to-failure as a function of maximum stress range. Such a procedure, however, fails to account for the large variation in the time-to-failure at a given maximum stress level. Fatigue data with statistical significance requires a large number of replicates at a given maximum stress level in order to measure the distribution of time-to-failure.

In this report a procedure which allows the generation of an S-N curve with some statistical value without resorting to an extremely large data base is explored in detail. The "wearout" or "strength degradation" model approach (References 1, 2, 3) provides one means for accomplishing this. Such an approach, however, involves the assumption of a direct relationship between static strength distribution, residual strength distribution after time under a specified load history, and distribution of time-to-failure at a maximum stress level. These types of models have to be carefully defined in terms of the load history to be applied to the material. For example, if the load history is tension/compression then the concept of strength degradation must consider both residual tension and residual compression, along with possible competing failure modes. The alternative approach to S-N curve characterization does not require any assumptions relating fatigue life to residual strength. As shown in the Appendix, the procedure does measure parameters which are completely compatible with the "wearout" model approach without any residual strength measurements.

SECTION II

S-N CURVE CHARACTERIZATION

An alternative to the "wearout" model approach for fatigue characterization of composite materials has been proposed by Hahn and Kim (Reference 4). Their approach involves two basic assumptions: (1) a classical power law representation of the S-N curve (Reference 5), and (2) a two parameter Weibull distribution of time-to-failure. In mathematical form these assumptions become

$$CNS^b = 1 \tag{1}$$

where S is the stress range, N is the number of cycles to failure, and b and C are material constants. In addition,

$$R\ (N) = \exp\left[-\left(\frac{N}{N_o}\right)^{\alpha_f}\right] \tag{2}$$

where R (N) denotes the reliability of N (probability of survival), N_o is the characteristic time-to-failure (location parameter), and α_f is the fatigue shape parameter. As illustrated in the Appendix, Equations 1 and 2 can also be derived from the "wearout" model approach with α_f being related to the shape parameter for static strength (Reference 3). Equation 1 can be written in the form

$$N = C^{-1}S^{-b} \tag{3}$$

substituting Equation 3 into Equation 2 and solving for S yields

$$S = K\left\{\left[-\ln R\ (N)\right]^{-\frac{1}{\alpha_f b}}\right\} N_o^{\ -1/b},\quad \alpha_f \neq f(S) \tag{4}$$

where

$$K = C^{-\frac{1}{b}} \tag{5}$$

When $N = N_o$, $-\ln R\ (N_o) = 1$ and Equation 4 reduces to

$$S(N_o) = K\ N_o^{-\frac{1}{b}} \tag{6}$$

A plot of log S versus log N_o produces a straight line with slope − 1/b and a y intercept of log K. Thus, a measure of the distribution of time-to-failure at various stress ranges in conjunction with Equation 2 allows α_f to be determined along with a set of values of N_o, each corresponding to a value of S. Equation 4 can then be utilized to produce an S-N_o curve for any desired reliability, R(N).

However, from a practical standpoint, one is more interested in obtaining an S-N curve for any desired level of reliability rather than an S-N_o curve. Writing Equation 6 in the form

$$N_o = C^{-1}S^{-b} \tag{7}$$

and substituting into Equation 2 yields

$$R(N) = \exp\left[-\left(\frac{N}{C^{-1}S^{-b}}\right)^{\alpha_f}\right] \tag{8}$$

Solving for S leads to the following S-N relationship for any desired level of reliability

$$S = K\left\{\left[-\ln R(N)\right]^{\frac{1}{\alpha_f b}}\right\}N^{-\frac{1}{b}} \tag{9}$$

SECTION III

DATA REDUCTION PROCEDURE

The data reduction procedure consists of:

(a) Fitting the time-to-failure data at each stress range to a two parameter Weibull distribution.

(b) Use a data pooling scheme to determine the fatigue shape parameter α_f.

(c) Fit log S versus log N_o data to a straight line for the determination of b and K.

Let m be the number of stress ranges tested and n_i the number of specimens tested at the i-th stress range, S_i, which leads to the data set

$$N_i\ (N_{i1}, N_{i2}, \ldots, N_{in_i}), \quad i = 1, 2, \ldots, m \tag{10}$$

Each stress range is fit to the two parameter Weibull distribution

$$R(N_i) = \exp\left[-\left(\frac{N_i}{N_{oi}}\right)^{\alpha_{fi}}\right] \tag{11}$$

A number of procedures can be utilized for determining α_{fi} and N_{oi}. One of the methods preferred by statisticians is the maximum likelihood estimator (MLE) which is of the form (Reference 6)

$$\frac{\sum_{j=1}^{n_i} N_{ij}^{\hat{\alpha}_{fi}} \ln N_{ij}}{\sum_{j=1}^{n_i} N_{ij}^{\hat{\alpha}_{fi}}} - \frac{1}{n_i}\sum_{j=1}^{n_i} N_{ij} - \frac{1}{\hat{\alpha}_{fi}} = 0 \tag{12}$$

$$\hat{N}_{oi} = \left(\frac{1}{n_i} \sum_{j=1}^{n_i} N_{ij}^{\hat{\alpha}_{fi}} \right)^{1/\hat{\alpha}_{fi}} \tag{13}$$

where $\hat{\alpha}_{fi}$ and $\hat{N}_{oi}$ denote estimated values of α_{fi} and N_{oi}, respectively. Equation 12 has only one real positive root. As a result, an iterative scheme can be utilized until a value of $\hat{\alpha}_{fi}$ is obtained to any desired number of decimal places. The resulting value of $\hat{\alpha}_{fi}$ obtained from the iterative scheme can then be used in conjunction with Equation 13 to obtain $\hat{N}_{oi}$.

Since fatigue shape parameters are estimated based on a particular sample size, it is anticipated that each value of S_i would produce a different value of $\hat{\alpha}_{fi}$, even though α_f may be independent of stress range. A two sample test (Reference 7) is available, however, which allows for testing the equality of shape parameters in two parameter Weibull distributions with unknown scale parameters. The approach is based on MLE and the results depend on sample size and confidence level desired. Let $\hat{\alpha}_{max}$ and $\hat{\alpha}_{min}$ be the maximum and minimum values obtained for $\hat{\alpha}_{fi}$. For the information tabulated in Reference 7, it is required that α_{max} and α_{min} be associated with equal sample sizes, n. If $\alpha_{f\ max}$ and $\alpha_{f\ min}$ are from the same distribution, then it is expected that (Reference 7)

$$\frac{\hat{\alpha}_{f\ max}}{\hat{\alpha}_{f\ min}} < \beta(\gamma\ ,\ n),\ \beta > 1 \tag{14}$$

for a given confidence level, α, and sample size, n. Values of β are shown in Table 1 for various sample sizes corresponding to a confidence level of 0.98. This data is taken from Reference 7. The large values of A associated with small sample sizes suggest that significant variations in $\hat{\alpha}_{fi}$ are likely to be encountered with small data sets taken from the same population.

Let us now assume that α_f is independent of stress range. Then a data pooling technique must be utilized in order to determine a single value of α_f for all S_i. Various approaches for obtaining a pooled value of α_f can be found in the literature. The approach used in the present work has been investigated by Lemon (Reference 8). This procedure utilizes the normalized data set

$$X\,(X_{i1}, X_{i2}, \ldots, X_{in_i}), \quad i = 1, 2, \ldots, m \tag{15}$$

where

$$X_{ij} = \frac{N_{ij}}{\hat{N}_{oi}} \tag{16}$$

Thus, each set of data at a given stress range is normalized by the estimated characteristic time-to-failure and the results fit to the pooled two parameter Weibull distribution

$$R\,(X) = \exp\left[-\left(\frac{X}{X_o}\right)^{\alpha_f}\right] \tag{17}$$

This procedure has the advantage of obtaining a large data base for determining α_f by using a few replicates for a number of values of S_i. In general, for equal accuracy fewer specimens are needed to determine the location parameter than shape parameter

For the pooled Weibull distribution, Equation 17, the MLE relationships take the form

$$\frac{\sum_{i=1}^{m}\sum_{j=1}^{n_i} X_{ij}^{\bar{\alpha}_f} \ln X_{ij}}{\sum_{i=1}^{m}\sum_{j=1}^{n_i} X_{ij}^{\bar{\alpha}_f}} - \frac{1}{M}\sum_{i=1}^{m}\sum_{j=1}^{n_i} X_{ij} - \frac{1}{\bar{\alpha}_f} = 0 \tag{18}$$

$$\bar{X}_o = \left(\frac{1}{M}\sum_{i=1}^{m}\sum_{j=1}^{n_i} X_{ij}^{\bar{\alpha}_f}\right)^{1/\bar{\alpha}_f} \tag{19}$$

where $\bar{\alpha}_f$ and $\bar{X}_o$ are estimated values of α_f and X_o, respectively, and

$$M = \sum_{i=1}^{m} n_i \tag{20}$$

For a perfect fit to the data pooling scheme, the location parameter, X_o, should be unity. The value of $\hat{N}_{oi}$ can be adjusted to produce an exact value of unity for X_o. In particular,

$$\bar{N}_{oi} = \bar{X}_o \hat{N}_{oi} \tag{21}$$

where $\bar{N}_{oi}$ denotes estimated values of N_o associated with the adjusted two parameter Weibull distribution

$$R (X) = \exp (-X^{\alpha_f}) \tag{22}$$

The slope of the S-N curve, l/b, and the y-intercept, K, can be determined by fitting log S_i versus log $\bar{N}_{oi}$ to a straight line. With K, b, and α_f now determined, Equation 9 can be used to produce an S-N curve of any desired reliability.

It should be noted that MLE is asymptotically unbiased, i.e., it is a biased estimator for small sample sizes (Reference 6). Unbiasing factors are tabulated in Reference 9. These factors are less than unity as MLE always tends to overestimate the shape parameter. Confidence intervals for both the shape parameter and location parameter as a function of sample size have also been established (Reference 9). If conservative estimates are desired for R (N), then a lower bound value of α_f can be utilized. This value of α_f can be used in conjunction with Equation 19 to determine $\bar{X}_o$.

SECTION IV

CENSORING PROCEDURES

In the case of high cycle fatigue the time-to-failure may become unacceptably long. This difficulty can be overcome by raising the stress range so that fatigue failures are produced within a reasonable number of cycles. For filament dominated laminates, which tend to have a very flat S-N curve, stress levels may have to be raised to an unacceptable level to produce a reasonable time-to-failure for all specimens tested. In particular, it is undesirable to raise the fatigue stress level to such a degree that it significantly overlaps the static strength distribution. In such cases the probability of a first cycle failure is significant.

If censoring techniques are applied to data reduction procedures, fatigue failures are not required of all specimens. For the data reduction scheme outlined in the present work, Type I censoring seems to be the most desirable in terms of yielding the most information. In the case of Type I censoring the fatigue test is terminated at a pre-determined time (e.g. 10^6 cycles) even though all specimens have not failed. The MLE equations for Type I censoring are of the form (Reference 6)

$$\frac{\sum_{j=1}^{r_i} N_{ij}^{\hat{\alpha}_{fi}} \ln N_{ij} + (n_i - r_i) R_i^{\hat{\alpha}_{fi}} \ln R_i}{\sum_{j=1}^{r_i} N_{ij}^{\hat{\alpha}_{fi}} + (n_i - r_i) R_i^{\hat{\alpha}_{fi}}} - \frac{1}{r_i} \sum_{j=1}^{r_i} \ln N_{ij} - \frac{1}{\hat{\alpha}_{fi}} = 0 \qquad (23)$$

$$N_{oi} = \left\{ \frac{1}{r_i} \left[\sum_{j=1}^{n_i} N_{ij}^{\hat{\alpha}_{fi}} + (n_i - r_i) R_i^{\hat{\alpha}_{fi}} \right] \right\}^{\frac{1}{\hat{\alpha}_{fi}}} \tag{24}$$

where n_i now denotes the total number of specimens tested at S_i, r_i is the number of fatigue failures at S_i, and R_i is the number of cycles at which the test is terminated.

The data pooling procedure is now analogous to "progressive censoring" in which a number of samples are removed at pre-determined time intervals throughout the duration of the test. The MLE associated with the data pooling procedure in conjunction with censored samples becomes

$$\frac{\sum_{i=1}^{m} \sum_{j=1}^{r_i} X_{ij}^{\bar{\alpha}_f} \ln X_{ij} + \sum_{i=1}^{m} (n_i - r_i) Y_i^{\bar{\alpha}_f} \ln y_i}{\sum_{i=1}^{m} \sum_{j=1}^{r_i} X_{ij}^{\bar{\alpha}_f} + \sum_{i=1}^{m} (n_i - r_i) Y_i^{\bar{\alpha}_f}} \tag{25}$$

$$- \frac{1}{N} \sum_{i=1}^{m} \sum_{j=1}^{r_i} X_{ij} - \frac{1}{\bar{\alpha}_f} = 0$$

$$\bar{X}_o = \left\{ \frac{1}{N} \left[\sum_{i=1}^{m} \sum_{j=1}^{r_i} X_{ij}^{\bar{\alpha}_f} + \sum_{i=1}^{m} (n_i - r_i) Y_i^{\bar{\alpha}_f} \right] \right\}^{\frac{1}{\bar{\alpha}_f}} \tag{26}$$

where

$$Y_i = \frac{R_i}{N_{oi}} \tag{27}$$

and N is the total number of fatigue failures, i.e.

$$N = \sum_{i=1}^{m} r_i \tag{28}$$

SECTION V

EXAMPLE DATA

Consider the tension-tension (T-T) fatigue data presented by Ryder and Walker (Reference 10) on quasi-isotropic T300/934[3] graphite-epoxy laminates with the stacking geometry $[45/90/-45/90-45/0/45/0]_S$ (laminate 2). Twenty replicates at three stress levels were used to characterize the scatter in time-to-failure. The effect of sample size on the Weibull parameters can be estimated by using a table of random numbers to select sample sizes of 5, 10, 15, and 20 for the three stress levels and calculating the resulting Weibull parameters. The results are shown in Table 2 along with the pooled parameters $\overline{X}_o$ and $\overline{\alpha}_f$. The Weibull parameters are determined by MLE. Note that the values of $\hat{N}_{oi}$ are not radically effected by sample size, while $\hat{\alpha}_{fi}$ is very sensitive to n. The same trend is noted for the pooled parameters $\overline{X}_o$ and $\overline{\alpha}_f$. It should also be noted that the data for n=20 satisfies the criterion of Equation 14 for data pooling.

Comparison between the normalized data and the Weibull distribution obtained from the pooling procedure is shown in Figure 1. Three additional stress levels with 3 replicates each are added to the data pooling process, yielding a total of 69 pooled data points in Figure 1. Comparisons between data and Weibull distributions obtained from the pooling procedure are shown in Figures 2-4 for the three stress levels with 20 replicates. In Figures 1-4, data points are converted to probabilities of survival from the Median Rank (MR) defined as

$$MR = \frac{j - 0.3}{n + 0.4} \qquad (29)$$

where j is the survival order number (data listed in decreasing order of time-to-failure) and n is the total number of samples tested. Equation 29 is an estimate of the reliability function R. When probability of failure is desired Equation 29 is applied with the data listed in increasing time-to-failure.

While the pooled data in Figure 1 compares favorably with the estimated two parameter Weibull distribution, similar data correlation for the individual stress levels in Figures 2-4 are much less favorable. This is to be anticipated as small data samples in conjunction with large scatter creates difficulty in fitting the data to any reasonable distribution function. These results are also a good illustration of the desirability of data pooling for obtaining a larger sample size for estimating shape parameters. The characteristic S-N curve resulting from the data reduction scheme is shown in Figure 5 along with a 95 percent survivability curve calculated from Equation 9. The numbers in parentheses correspond to the sample size associated with a specific stress level. Scatter bands on time-to-failure are also shown. The solid dots correspond to fatigue failures outside the 95 percent survivability line. The pooled Weibull parameters along with the S-N curve parameters are listed in Table 3.

The S-N characterization procedure has also been applied to other data available in the literature and the results are shown in Figures 6-8. As in Figure 5, sample size at each stress level, scatter bands, and fatigue failures outside of the 95 percent survivability line are shown.

In Figure 6 tension-compression (T-C) constant amplitude fatigue data from laminate 2 is illustrated in Reference 10. Data in Figure 7 is also from the work of Ryder and Walker (Reference 11). The same graphite-epoxy material as in laminate 2 is utilized for this T-T data with the stacking geometry $[0/+45/0_2/-45/0_2/+45/0_2/-45/0]_s$. This composite is designated laminate 1 and presents a very interesting example for the recommended fatigue characterization procedure. Because of the flat nature of the S-N curve, fatigue failures were difficult to induce. For the split numbers in parentheses, the first number designates the total number of samples tested at that stress level and the second number refers to the number of fatigue failures recorded. Thus, this data had to be reduced utilizing the censoring procedure outlined. All tests were terminated after 10^6 cycles. It is interesting to note the extremely low value of $\overline{\alpha}_f$ associated with this laminate. This behavior has been previously noted for filament dominated laminates (Reference 12) (laminates containing a large percentage of 0-degree plies relative to the load direction). The straight line fit of the S-N curve is not as good for this laminate as for laminate 2 in Figures 5 and 6. The low number of fatigue failures along with the large scatter are likely reasons for the poorer fit.

The data in Figure 8 is taken from the work of Yang (Reference 13). This constant amplitude T-T data are for a $[\pm 45]_{2S}$ T300/5208 graphite-epoxy laminate. This orientation was chosen because it induces significant shear stress relative to the fiber direction in each ply. Since there are no 0-degree plies relative to the load direction, this laminate is referred to as matrix dominated. This composite is designated laminate 3. The laminate numbers have been assigned in descending order of filament dominance. It is interesting to note the relatively high value of $\bar{\alpha}_f$ associated with laminate 3 compared to laminates 1 and 3. Because of the higher value of the shape parameter, the amount of shift of the 95 percent survivability line down from the characteristic life line is much less compared to laminates 1 and 2 in Figures 5-7.

To demonstrate the basic difference in S-N curve behavior for constant amplitude T-T loading, the 95 percent survivability lines for the three laminates under consideration are shown in Figure 8. Graphite fibers are essentially fatigue insensitive. Thus, filament dominated laminates tend to have a flatter S-N curve.

Pooled Weibull parameters along with the S-N curve parameters are shown in Table 3 for all laminates and loading conditions under consideration. Note that none of the values for $\bar{\alpha}_f$ have been corrected for the bias of MLE. For these pooled sample sizes the bias correction is very small.

All stress ranges in Figures 5-9 are normalized by the characteristic static strength $\bar{S}_0$ obtained by fitting the static strength data to a two parameter Weibull distribution.

SECTION VI

CONCLUSIONS

A procedure has been outlined for characterizing the S-N behavior of composite laminates with some degree of statistical reliability. The data pooling procedure used in conjunction with the data reduction scheme offers the advantage of a large data base for determining a fatigue shape parameter without requiring large sample sizes at each stress range considered. Use of censoring techniques provides data reduction procedures without requiring all specimens to produce fatigue failures. Data presented shows the proposed characterization scheme to be promising.

If the proposed procedure is to be utilized, the fatigue experiments should be planned accordingly. In particular, it would be desirable to use the same sample size for all stress ranges tested. If 60 samples are desired for determining a pooled shape parameter, 10 replicates at 6 different stress ranges is preferable to 20 replicates at 3 different stress ranges. In the former case, 6 data points are available for determining the characteristic S-N curve, rather than 3 as would be provided by the latter case.

In the data reduction procedure, use of MLE is strongly recommended for determining Weibull parameters. Confidence intervals and other statistical tools based on MLE have been well established. Furthermore, the data pooling technique for censored data is based on MLE in conjunction with the concept of "progressive censoring."

It should also be noted that the procedure outlined can easily be revised to include S-N relationships other than the power law described by Equation 1.

APPENDIX

For simplicity, consider the case of constant amplitude tension-tension loading. If one assumes a crack growth model as the basis for a "strength degradation" model, the governing equation for residual strength takes the form

$$\frac{dF}{dn} = -\frac{AS^b}{c} F^{(1-c)} \tag{30}$$

where F is the strength after n cycles and A, b, and c are constants. Thus, there are three parameters which characterize residual strength degradation in this model. For the assumption that the flow growth is driven by a square root singularity at the crack tip (Reference 1)

$$b = 2\,(1-c) \tag{31}$$

and the number of parameters is reduced to two.

Separating variables in Equation 30 yields

$$\int_{F_o}^{F} y^{(1-c)} dy = -AS^b \int_o^n dx \tag{32}$$

Performing the integration leads to the residual strength equation

$$F_o^{\,c} = F^c(n) + AS^b n \tag{33}$$

where F_o is the static strength. Let us assume that the static strength is described by the two parameter Weibull distribution

$$R(F_o) = \exp\left[-\left(\frac{F_o}{S_o}\right)^{\alpha_o}\right] \tag{34}$$

where S_o is the location parameter of the static strength distribution and α_o is the static strength shape parameter. Substituting Equation 33 into Equation 34 yields

$$R(F) = \exp\left[-\left(\frac{F^c + AS^b n}{S_o^{\,c}}\right)^{\alpha_o/c}\right] \tag{35}$$

Denoting the number of cycles to fatigue failure by N_f, it is assumed that failure actually occurs when the stress reaches its maximum value during the fatigue cycle. In terms of residual strength this simply states that the residual strength is reduced to S one cycle prior to failure, as failure occurs on the next cycle when loaded to S. Stated mathematically,

$$F(n) = S, \quad n = N_f - 1 = N \tag{36}$$

and Equation 33 becomes

$$F_o^{\,c} = S^c + AS^bN \tag{37}$$

The case N = 0 corresponds to a one cycle failure and Equation 37 becomes

$$S = F_o$$

which simply implies that a first cycle failure is a special case where the maximum stress in the fatigue cycle corresponds to the static strength.

Using Equation 37 in conjunction with Equation 34 leads to the following reliability function for time-to-failure

$$R(N) = \exp\left[-\left(\frac{N + A^{-1}S^{c-b}}{A^{-1}S^{-b}S_o^{\,c}}\right)^{\alpha_f}\right] \tag{39}$$

where α_f is the fatigue shape parameter

$$\alpha_f = \alpha_o / c \tag{40}$$

Equation 39 can be written in the form of a three parameter Weibull distribution

$$R(N) = \exp\left[-\left(\frac{N - L_o}{N_o - L_o}\right)^{\alpha_f}\right] \tag{41}$$

where

$$N_o = S_o^{\,c}A^{-1}S^{-b}\left[1 - \left(\frac{S}{S_o}\right)^c\right] \tag{42}$$

$$L_o = -A^{-1}S^{c-b} \tag{43}$$

The three parameter Weibull distribution is simply a shifted two parameter Weibull distribution. It should be noted that L_o is negative as A and S are positive numbers. In the present context, this parameter does not have the connotation of a minimum life. It is, in fact, a statement of the probability of a first cycle failure. In particular, if N = 0 Equation 41 reduces to the static distribution, Equation 34

Writing Equation 42 in the form

$$CS^b N_o \doteq \left[1 - \left(\frac{S}{S_o}\right)^c\right] \tag{44}$$

with $C = AS_o^{\ c}$, yields the basic form of the S-N curve which is different than Equation 1. It should be noted, however, that the parameter c is simply a ratio of the static strength shape parameter to the fatigue shape parameter. Experience has shown this number to be typically greater than 10 for composites. In addition, the stress levels, S, of concern are considerably less than the static strength location parameter, S_o. Thus,

$$\left(\frac{S}{S_o}\right)^c << 1 \tag{45}$$

and Equation 44 becomes

$$CS^b N_o = 1 \tag{46}$$

which is of the same form as Equation 7. Thus, the "wearout" model approach yields an S-N curve, for stress levels of interest, which is of the same form as assumed for the proposed data reduction procedure.

It should be noted that the "strength degradation" model proposed by Yang and Liu (Reference 3) is not based on a flaw growth law. Instead they assume

$$AS^b = f(S, \omega\ r) \tag{47}$$

where ω is the frequency and r the stress ratio. They are forced to assume the shape of the S-N curve in order to determine f. The relationship assumed was of the same form as Equation 46. Thus, from a mathematical standpoint their model is the same as the crack growth or "wearout" model.

Consider Equation 39 in the form

$$R(N) = \exp\left\{ - \left[\frac{N}{N_o} + \left(\frac{S}{S_o}\right)^{c} \right]^{\alpha_f} \right\} \tag{48}$$

where

$$N_o = A^{-1} S_o^{\,c} S^{-b} \tag{49}$$

For high cycle fatigue the second term in the exponential is negligible and R(N) can be approximated by the two parameter Weibull distribution

$$R(N) = \exp\left[- \left(\frac{N}{N_o}\right)^{\alpha_f} \right] \tag{50}$$

Thus, the data reduction procedure proposed measures parameters which are consistent with "wearout" models or "strength degradation" models. Such models must be modified from the details presented here if more complex loadings other than T-T are to be considered. Competing failure modes which may interact are a problem in the case of T-C, for example.

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TABLE 1

EQUALITY OF WEIBULL SHAPE PARAMETERS,
α = 0.98 (REFERENCE 7)

n_i	β
5	3.550
10	2.213
15	1.870
20	1.703
100	1.266

TABLE 2

EFFECT OF SAMPLE SIZE ON WEIBULL PARAMETERS

	STRESS LEVEL, MPa (KSI)							
	290(42)		345(50)		400(58)			
n	$\hat{\alpha}_{f1}$	$\hat{N}_{o1}$	$\hat{\alpha}_{f2}$	$\hat{N}_{o2}$	$\hat{\alpha}_{f3}$	$\hat{N}_{o3}$	$\bar{\alpha}_f$	$\bar{X}_o$
5	3.05	1,865,519	44.7	70,199	0.804	3,116	1.53	1.090
10	1.85	1,485,006	1.14	91,949	1.05	3,803	1.23	1.007
15	1.47	2,106,780	1.23	87,407	0.858	3,206	1.13	1.011
20	1.51	1,964,890	1.07	100,815	0.984	4,042	1.14	1.007

TABLE 3

S-N CURVE AND POOLED WEIBULL PARAMETERS

Laminate	b	K/S_o	$\bar{\alpha}_f$	$\bar{X}_o$
1(T-T)	66.34	1.066	0.310	0.9490
2(T-T)	21.70	1.188	1.08	1.014
2(T-C)	9.705	1.791	1.45	0.9863
3(T-T)	65.84	1.094	3.82	1.003

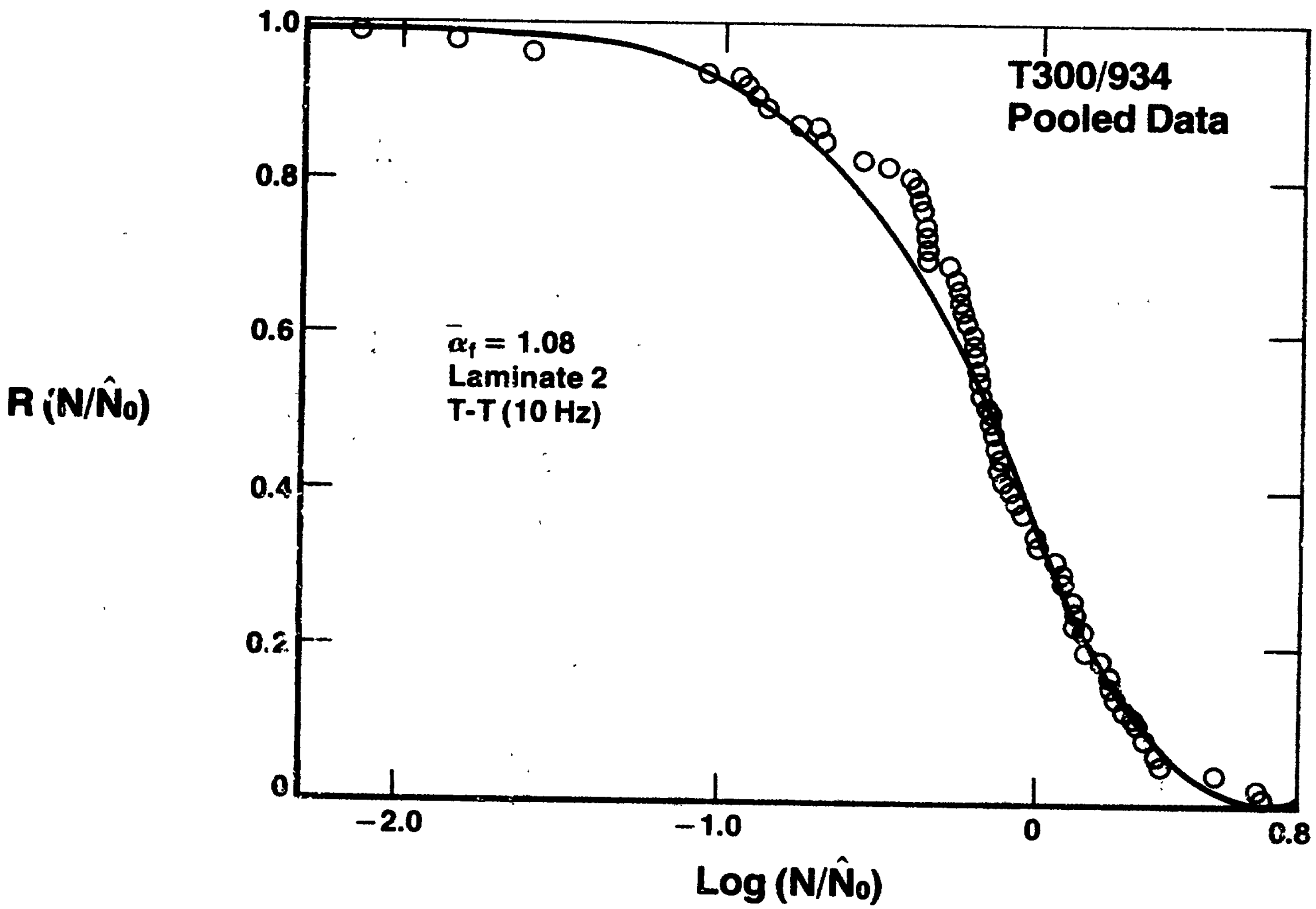

Figure 1. Comparison Between Normalized Data and Pooled Two Parameter Weibull Distribution

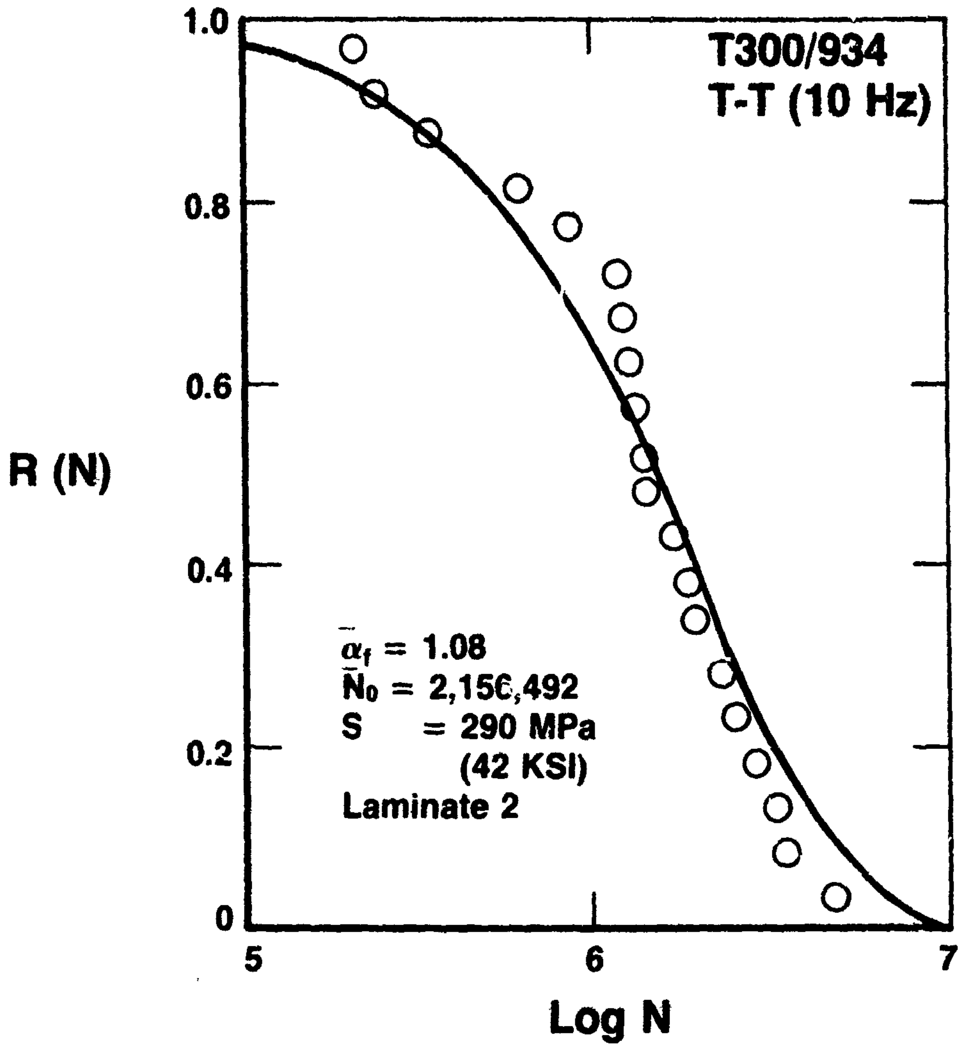

Figure 2. Comparison Between Data and Pooled Two Parameter Weibull Distribution $S/\bar{S}_o = 0.60$

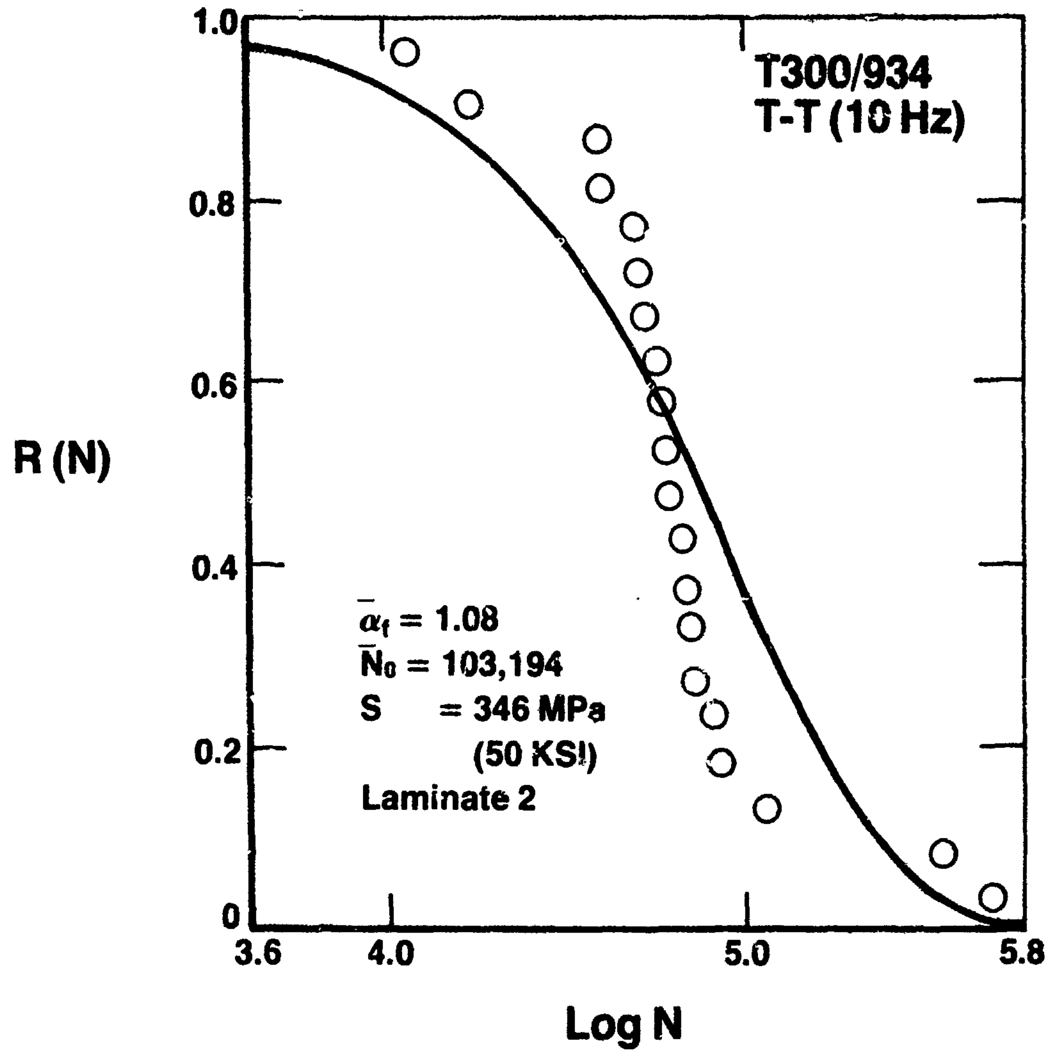

Figure 3. Comparison Between Data and Pooled Two Parameter Weibull Distribution $S/\bar{S}_o = 0.71$

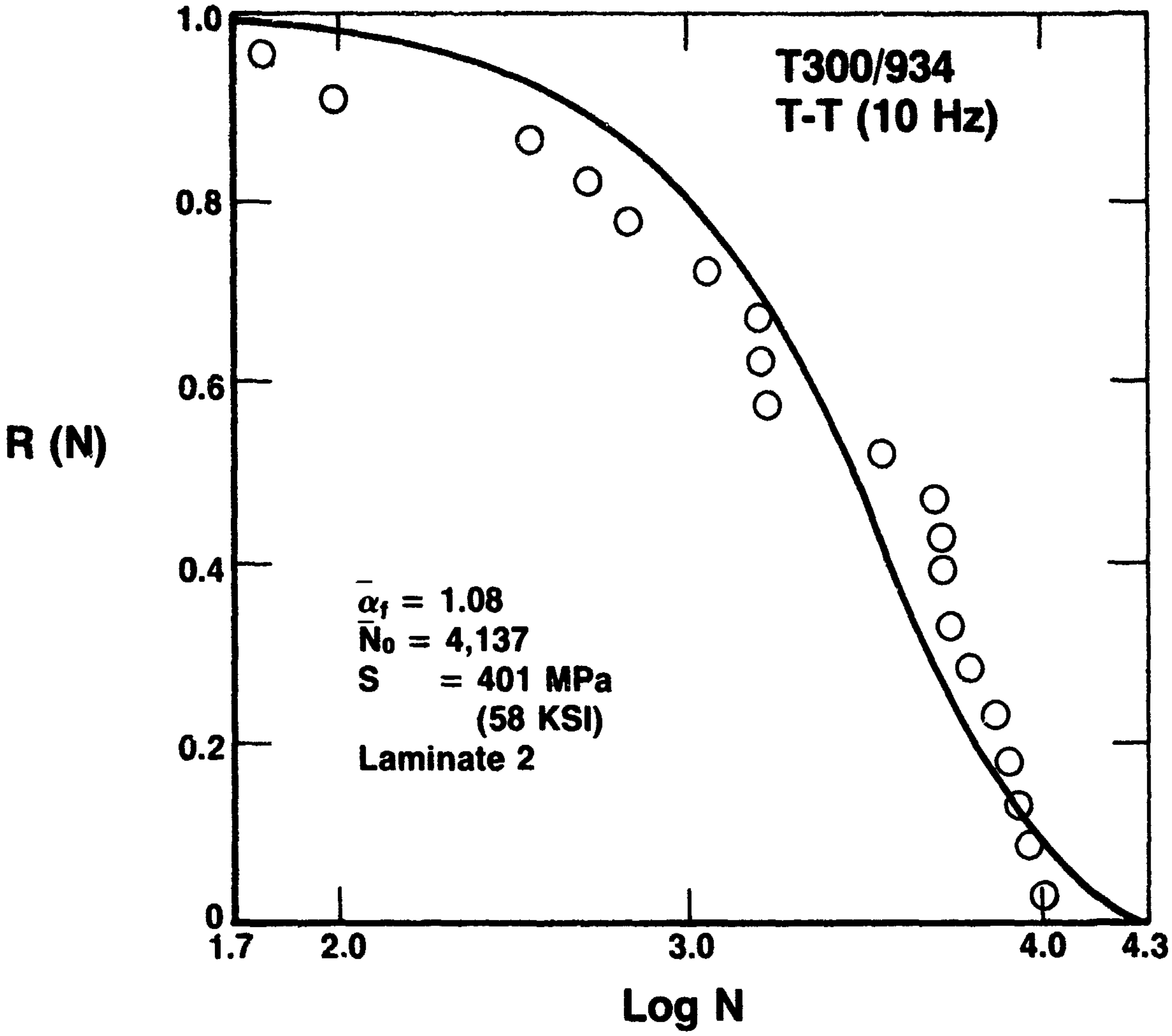

Figure 4. Comparison Between Data and Pooled Two Parameter Weibull Distribution $S/\bar{S}_o = 0.82$

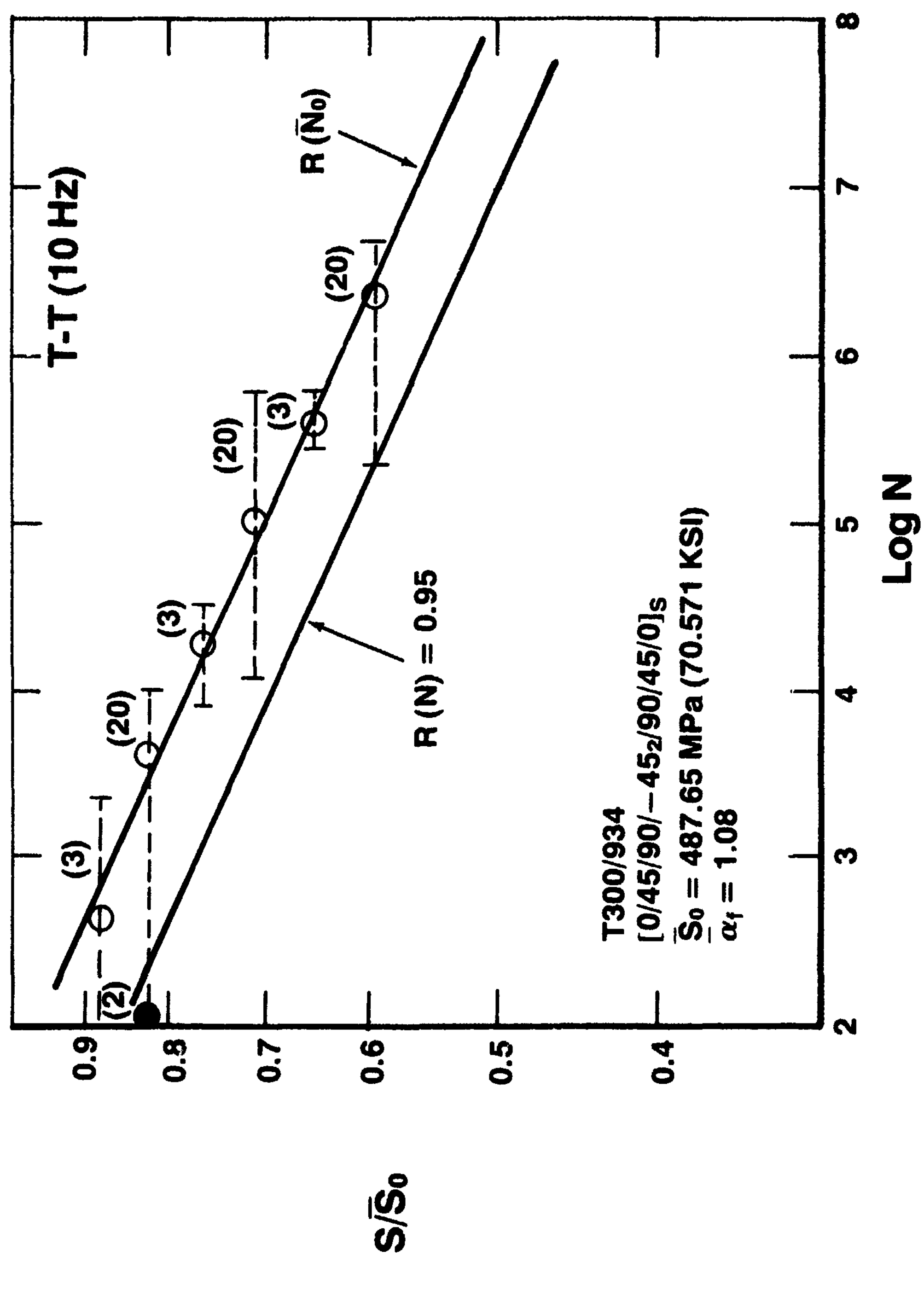

Figure 5. Tension - Tension S-N Curves for Laminate 2

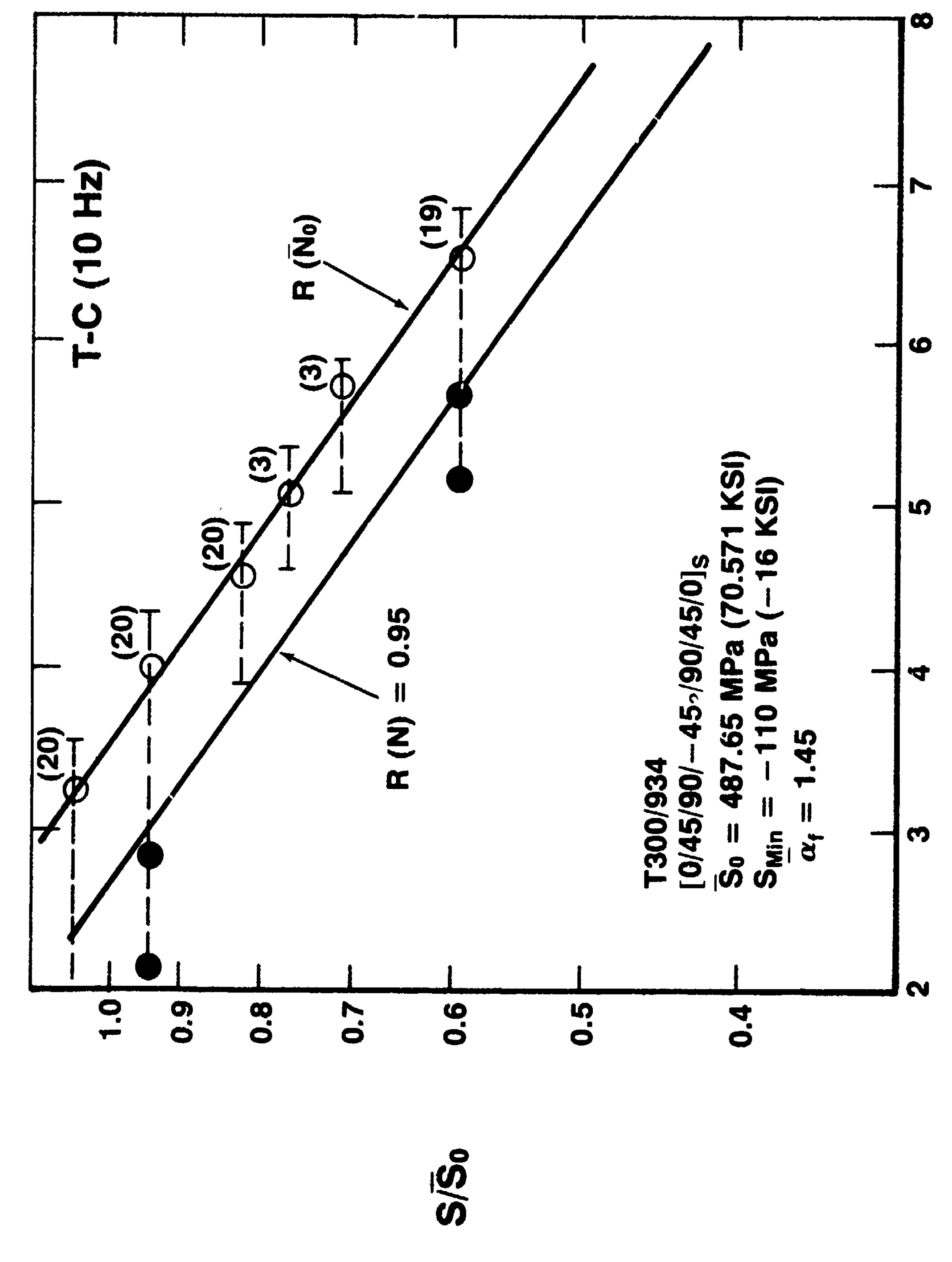

Figure 6. Tension-Comprehension S-N Curves for Laminate 2

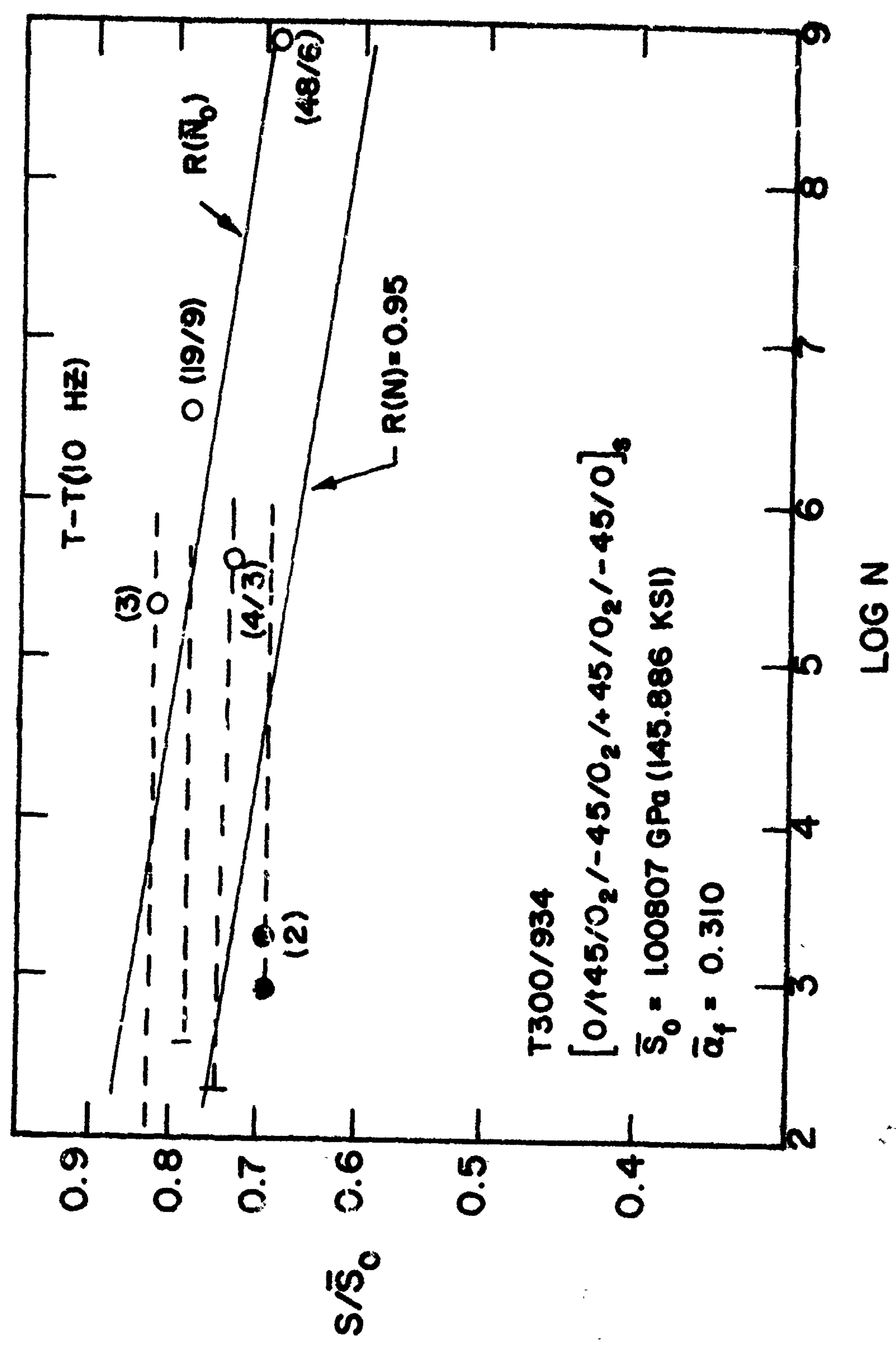

Figure 7. Tension-Tension S-N Curves for Laminate 1

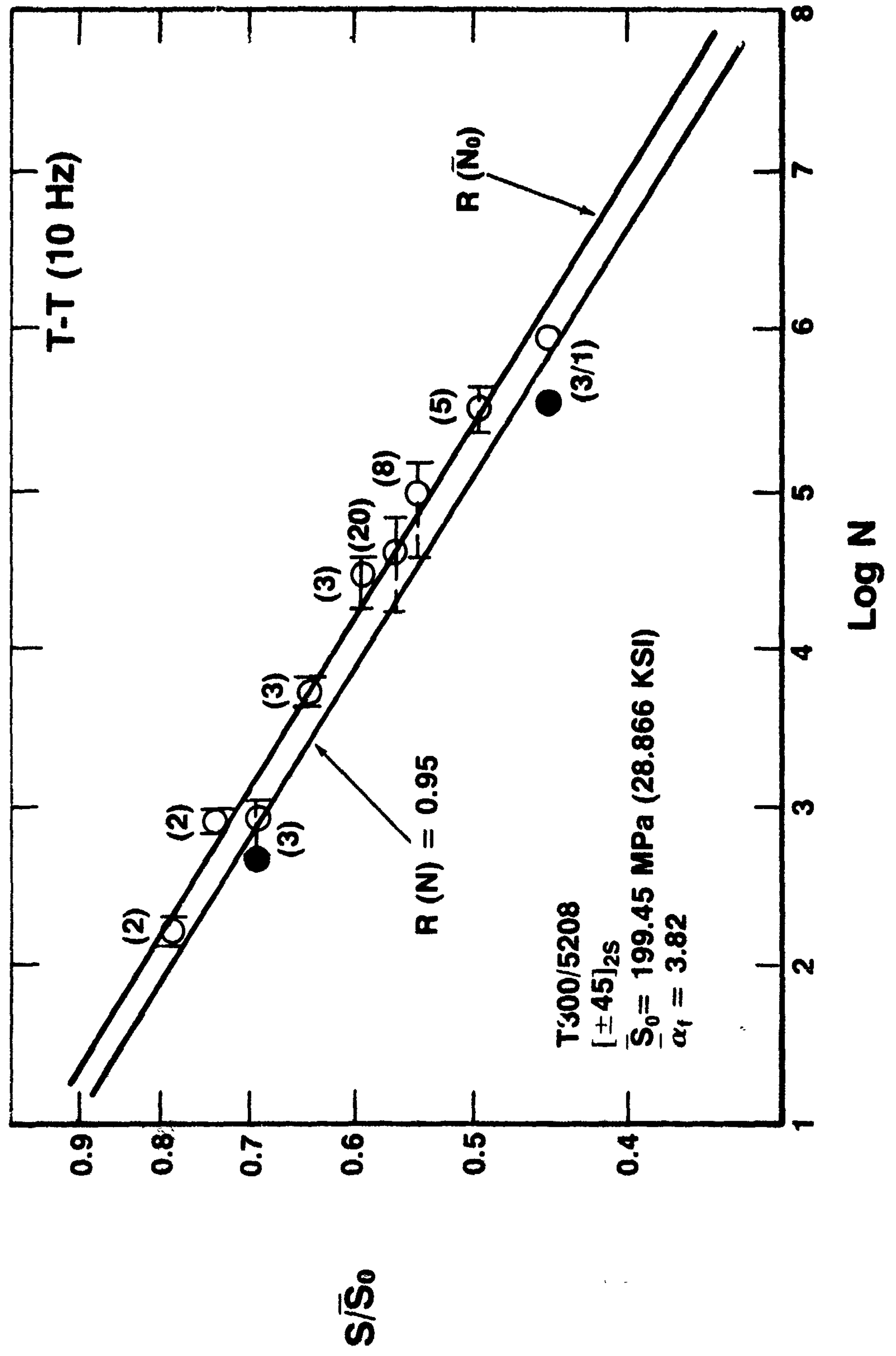

Figure 8. Tension-Tension S-N Curves for Laminate 3

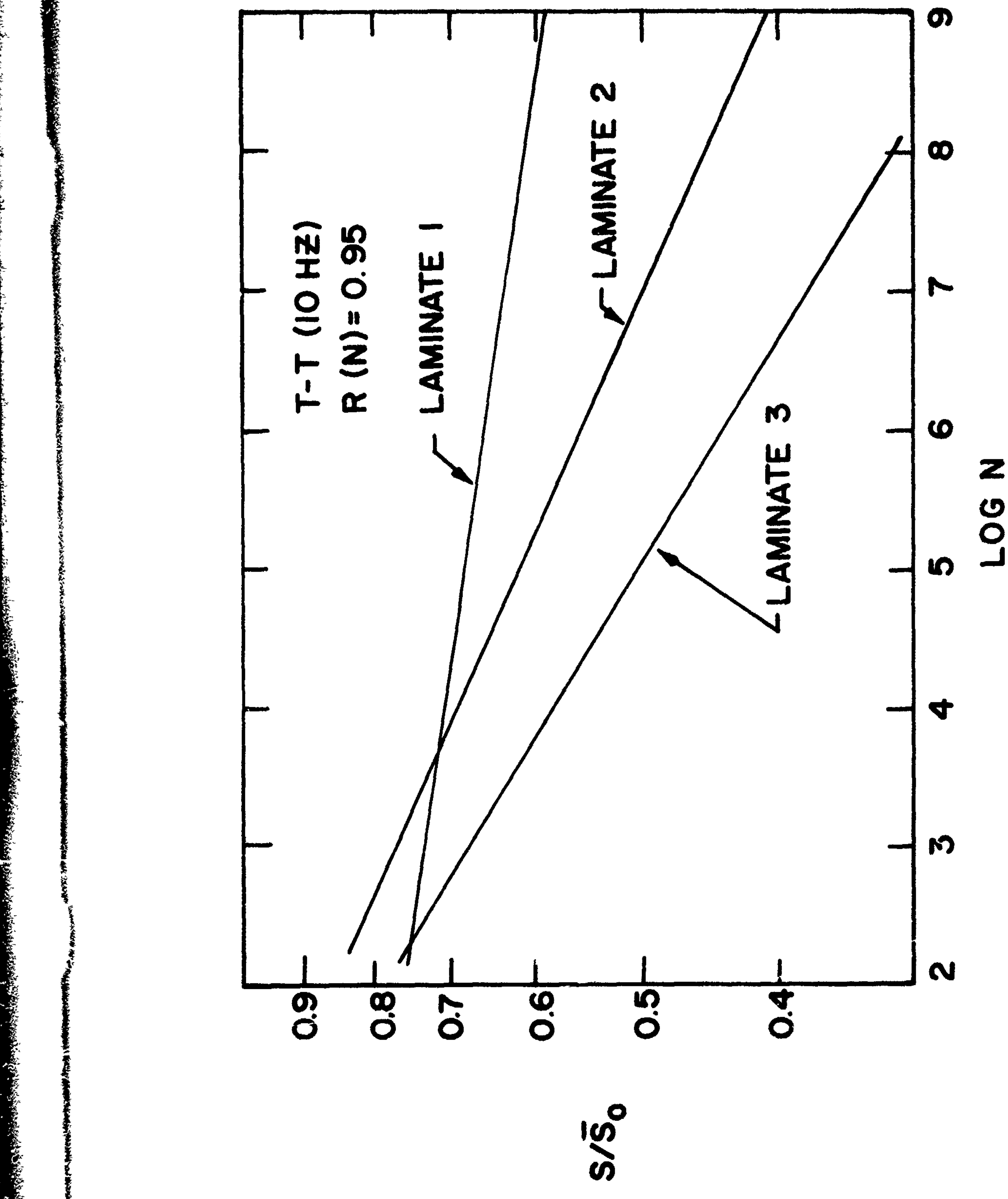

Figure 9. Comparison of 95 Percent Survivability S-N Curves for Tension-Tension Loading

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